AN FDTD IMPEDANCE BOUNDARY CONDITION AND ITS APPLICATION TO WAVEGUIDE DISCONTINUITY ANALYSES

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Abstract

A new method for calculating the scattering parameters of metal waveguide structures based on the FDTD field simulation will be presented. In this method, a type of the boundary condition called the Impedance Boundary Condition (IBC) is adopted at the circuit terminating ports. It allows us to excite the circuit with an arbitrary mode and terminate all ports by well-defined impedances. This implementation enables us to calculate the scattering parameters efficiently and rigorously.

I. Introduction

When we analyze a circuit in the time domain, it is necessary to terminate the circuit ports by an appropriate condition. The Absorbing Boundary Condition (ABC), which minimizes reflection of waves outgoing from the circuit, has been commonly used for this purpose at the boundary of analysis region in FDTD simulations. Then the S-parameters of the circuit are usually calculated as the ratios of reflected or transmitted signal spectrum to the incident one over a wide range of frequencies. Though this way of calculation is well acknowledged and is the most popular, it includes two potential delicate procedures: the troublesome procedure to separate the backward- and forward-going waves at the input port; and the complicated ABC calculation to minimize undesirable reflection from the boundary. These procedures require additional computational efforts, which are not minor ones.

In this paper we will present an alternative way of S-parameter calculation that is usable for metal waveguide circuit structures. One can avoid conventional procedures and can accomplish more efficient and rigorous S-parameter calculation.

II. Method

Although the condition of non-reflection at the circuit ports is useful when we are interested in the signal waveform, it is by no means the only requirement for S-parameter calculation. A more general requirement is that the ports should be terminated by well-defined impedances. In this sense, the ABC is a particular case that the ports are terminated by the characteristic impedance of the waveguide which is frequency dependent in general. Here, we will think about a termination that is expressed by a simple equivalent circuit. The method described here is an extension of [1]. Microstrip and coplanar circuits were analyzed using lumped element termination in [1]. However, in the case of metal waveguide structures, a more rigorous formulation is given to take into account of distinction of the modes.

II-I. Modal Voltage and Current

Let us consider a rectangular waveguide shown in Fig. 1 as an example. The cross sectional shape of the waveguide can actually be arbitrary. We can also have any kind of discontinuity that can be treated in conventional FDTD simulations between the ports. However, we need to assume homogeneous structure on the port planes, that is, the waveguide must be filled with a constant media at least at the cross section of the ports. Then the waveguide modes which

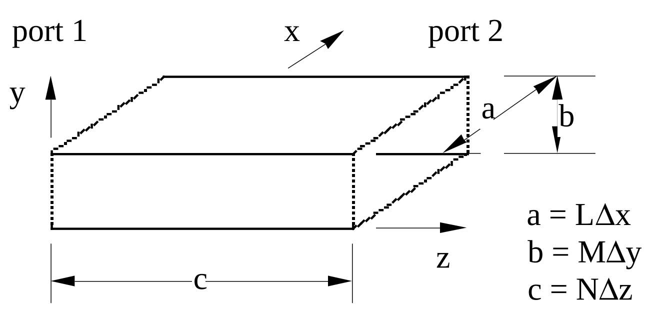

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Figure 1 Rectangular Waveguide

have field profiles not frequency-dependent can be defined on each port plane, and infinite number of modal ports can be considered at each physical port. Consequently, the "voltage V" and "current I" at each modal port can be defined using the following definitions.

$$E_t(x,y,t) = V(t)\, e_t(x,y) \qquad (1\text{-}1)$$

$$H_t(x,y,t) = I(t)\, h_t(x,y) \qquad (1\text{-}2)$$

Where, $e_t(x,y)$ and $h_t(x,y)$ are the unit modal eigenfunctions of transverse electric and transverse magnetic components normalized by

$$\int\int |e_t|^2 dS = \int\int |h_t|^2 dS = 1 \qquad (2)$$

respectively, and $E_t(x,y,t)$ and $H_t(x,y,t)$ are the modal field components of actual field. The integral in eq. (2) is taken over each port plane and the sign is determined such that

$$e_t = h_t \times \mathbf{z} \qquad (3)$$

holds, where $\mathbf{z}$ is the unit vector normal to the port plane oriented toward outside the circuit. It should be noticed here that eqs. (1-1) and (1-2) are time domain definitions and are valid only when the eigenfunctions e_t and h_t are frequency independent. These expressions are obtained by taking Fourier transform of corresponding frequency domain definitions which are found in standard textbooks on microwave circuits. The above definitions mean that the modal fields are also proportional to the modal eigenfunctions in time domain and observed total field can be expressed by a linear combination of the modal eigenfunctions at any moment. Therefore, we can calculate each modal voltage and current at any FDTD time step from observed field on the port planes using orthogonality between the modes.

II-II. Impedance Boundary Condition (IBC)

Since we have defined the voltage and current at the modal ports, we can now consider the terminating impedance. To do this, we need to go back in the frequency domain. Suppose that each port can be terminated by a certain impedance $Z_{term}(\omega)$. Then the following relationship should hold.

$$\tilde{V}(\omega)\, e_t(x,y) = Z_{term}(\omega)\, \tilde{I}(\omega)\, h_t(x,y) \qquad (4)$$

Here, ~ means that the variable is a phaser, i.e. a complex function, which is the Fourier transform of corresponding time domain variable. Referring to eqs. (1) to (3), we can rewrite eq. (4) using modal field Cartesian components as:

$$\tilde{E}_x(x,y,\omega) = \mp Z_{term}(\omega)\, \tilde{H}_y(x,y,\omega); \qquad (5\text{-}1)$$

$$\tilde{E}_y(x,y,\omega) = \pm Z_{term}(\omega)\, \tilde{H}_x(x,y,\omega), \qquad (5\text{-}2)$$

where - in (5-1) and + in (5-2) are taken for all modal ports at port 1, and + in (5-1) and - in (5-2) at port 2. This condition can be implemented in the FDTD algorithm as follows.

When we descretize fields using Yee's mesh, we have two choices in setting the nodes on each physical port of the structure shown in Fig. 1. The first choice is to adjust the mesh such that E_x-, E_y-, and H_z-nodes exist on the port plane. This case is illustrated in the left-hand side of Fig. 2. The second is such that H_x-, H_y-, and E_z-nodes come on the plane, which is shown in the right-hand side of Fig. 2. In the first case, the condition of a type:

$$Y_{term}(\omega) \equiv \frac{1}{Z_{term}(\omega)} = G_{term} + j\omega C_{term} \qquad (6)$$

can be implemented easily. Figure 3 shows an E_x-node and related neighboring H-nodes. The E_x value can usually be updated using previous values of these H-nodes. However, when the E_x-node exists on the plane of physical port 1, one of the H-nodes, i.e. $H_y(i+1/2,j,k-1/2)$ in Fig. 3, is nonexistent. Instead of this nonexistent value,

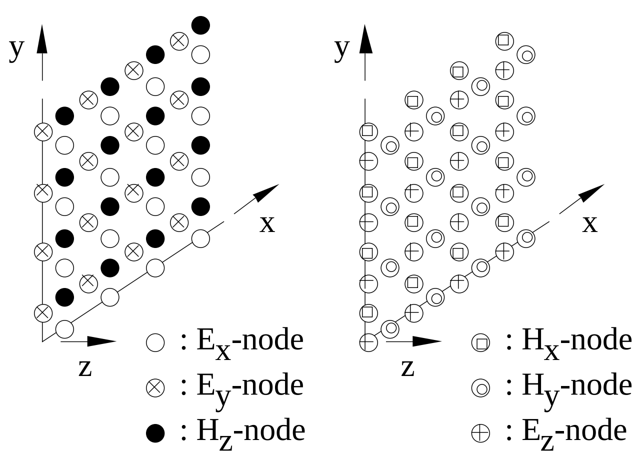

Figure 2 FDTD Nodes On The Port

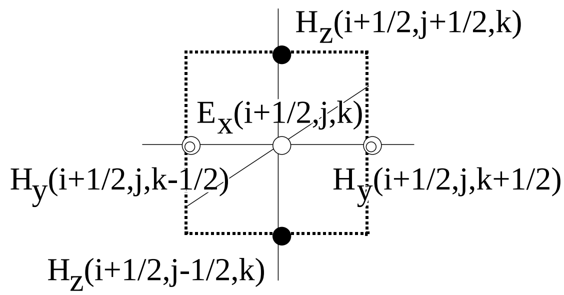

Figure 3 E_x-node And Its Vicinity

the condition of eq. (5-1) can be used. Substituting eq. (6) into eq. (5-1), one can rewrite this condition into the following time domain finite difference form.

$$\mp \frac{H_y^{n-1/2}(i+1/2,j,k-1/2) + H_y^{n-1/2}(i+1/2,j,k+1/2)}{2}$$
$$= G_{\mathrm{term}} \frac{E_x^{n}(i+1/2,j,k) + E_x^{n-1}(i+1/2,j,k)}{2}$$
$$+ C_{\mathrm{term}} \frac{E_x^{n}(i+1/2,j,k) - E_x^{n-1}(i+1/2,j,k)}{\Delta t} \qquad (7)$$

Notice here that we used a relationship $j\omega = \partial/\partial t$, and took an average in space for H_y and an average in time for E_x to evaluate the values at the point $[(i+1/2)\Delta x, j\Delta y, k\Delta z]$ and at the time $(n-1/2)\Delta t$. Substituting eq. (7) into the term of $H_y(i+1/2, j,k-1/2)$ in usual FDTD E_x-node updating formula, we get a new formula for updating the E_x-nodes on the physical port 1.

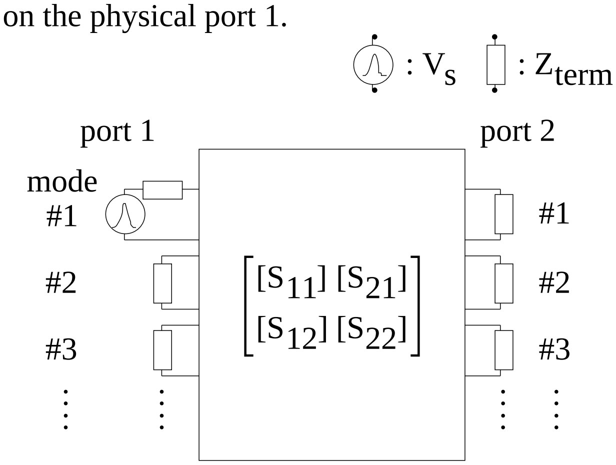

Figure 4 Equivalent Circuit Model

$$E_x^{n}(i+1/2,j,0) = \frac{\varepsilon_0\varepsilon_{rx} - \Delta t G_{\mathrm{term}}/\Delta z + 2C_{\mathrm{term}}/\Delta z}{\varepsilon_0\varepsilon_{rx} + \Delta t G_{\mathrm{term}}/\Delta z + 2C_{\mathrm{term}}/\Delta z} E_x^{n-1}(i+1/2,j,0)$$
$$+ \frac{2\Delta t}{(\varepsilon_0\varepsilon_{rx} + \Delta t G_{\mathrm{term}}/\Delta z + 2C_{\mathrm{term}}/\Delta z)\,\Delta z} H_y^{n-1/2}(i+1/2,j,1/2)$$
$$- \frac{\Delta t}{(\varepsilon_0\varepsilon_{rx} + \Delta t G_{\mathrm{term}}/\Delta z + 2C_{\mathrm{term}}/\Delta z)\,\Delta y} [\, H_z^{n-1/2}(i+1/2,j+1/2,0) - H_z^{n-1/2}(i+1/2,j+1/2,0) \,]$$
$$- \frac{2\Delta t}{(\varepsilon_0\varepsilon_{rx} + \Delta t G_{\mathrm{term}}/\Delta z + 2C_{\mathrm{term}}/\Delta z)\,\Delta z} [\, G_{\mathrm{term}} V_S^{n-1/2} + C_{\mathrm{term}} V'^{\,n-1/2}_S \,]\, e_x(x,y) \qquad (8)$$

In eq. (8), an excitation term in terms of voltage signal source has also been added. Notice here that the applied voltage V_s and its derivative V'_s are defined at each $(n-1/2)\Delta t$. Similar formula can be deduced for the E_y-nodes. By applying these formulas to all E_x- and E_y-nodes on the physical port 1, one particular mode can be excited whereas all modal ports on this physical port can be terminated by the condition eq. (6). Consequently, we finally get an equivalent circuit shown in Fig. 4 for the structure in Fig. 1. All circuit parameters can be deduced form this equivalent circuit model.

If the mesh setting described in the right-hand side of Fig. 2 is used, an impedance type of

$$Z_{\mathrm{term}}(\omega) = R_{\mathrm{term}} + j\omega L_{\mathrm{term}} \qquad (9)$$

can be implemented similarly.

II-III. S-parameter Calculation

The waveguide discontinuity can generally be characterized using a generalized scattering matrix. This matrix consists of submatrices which have ideally infinite but actually small number of dimensions corresponding to the number of modes to be considered at each physical port. These matrix elements, for instance S-parameters from port 1 mode #1 to any other modal ports, are calculated using the equivalent circuit model described in Fig. 4 as:

$$S_{11,\ \text{mode \#1}\leftarrow\text{mode \#1}} = \frac{2\tilde{V}_{1,\ \text{mode \#1}} - \tilde{V}_s}{\tilde{V}_s}, \quad (10\text{-}1)$$

$$S_{11,\ \text{mode \#i}\leftarrow\text{mode \#1}} = \frac{2\tilde{V}_{1,\ \text{mode \#i}}}{\tilde{V}_s} \quad (10\text{-}2)$$

for i = 2, 3, ..., and

$$S_{21,\ \text{mode \#i}\leftarrow\text{mode \#1}} = \sqrt{\frac{R_{term1}}{R_{term2}}}\frac{2\tilde{V}_{2,\ \text{mode \#i}}}{\tilde{V}_s}$$

for i = 1, 2, 3, ..., (10-3)

where Z_{term} is set to R_{term1} (or $1/G_{term1}$) at the physical port 1 and to R_{term2} (or $1/G_{term2}$) at the port 2, respectively. $\tilde{V}_s$ is Fourier transform of applied voltage at the port 1 mode #1, and $\tilde{V}_{1,\ \text{mode \#i}}$ and $\tilde{V}_{2,\ \text{mode \#i}}$ are that of calculated modal voltage at each corresponding modal port. Needless to say, these parameters are equivalent to the voltage wave scattering parameters when the all ports are terminated by R_{term} ($\equiv R_{term1} = R_{term2}$). We can easily convert these parameters into that of waveguide characteristic impedance termination after obtaining all matrix elements.

III. Numerical example

Figure 5 shows a rectangular waveguide having an inductive post. The S-parameters between the reference planes T and T' of this structure were calculated using the presented method. The first three possible modes, i.e., TE10, TE30 and TE50, are taken into account in this case and the results compare with those of an analytical equivalent circuit approach [2] as shown in Fig. 6.

This method is much more efficient than the conventional FDTD simulations and the result does not include an error caused from imperfection of ABCs. Furthermore, we do not have to worry about the separation of waves because the S parameters are derived directly from nodal voltages of the equivalent circuit. The calculation is almost automatic.

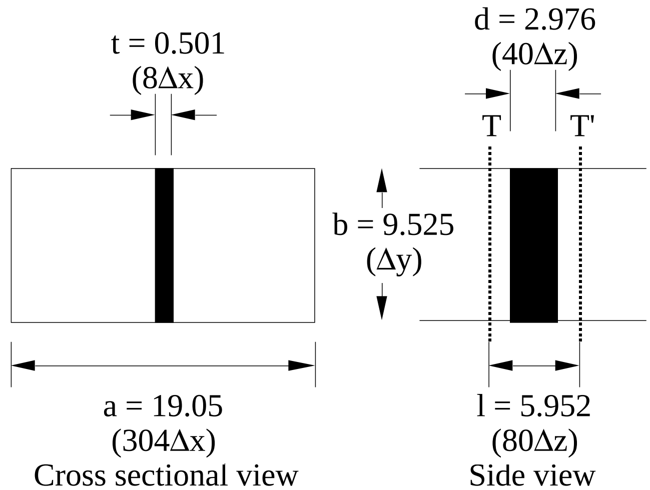

Figure 5 Inductive Post In Rectangular Waveguide Dimensions Are All In (mm)

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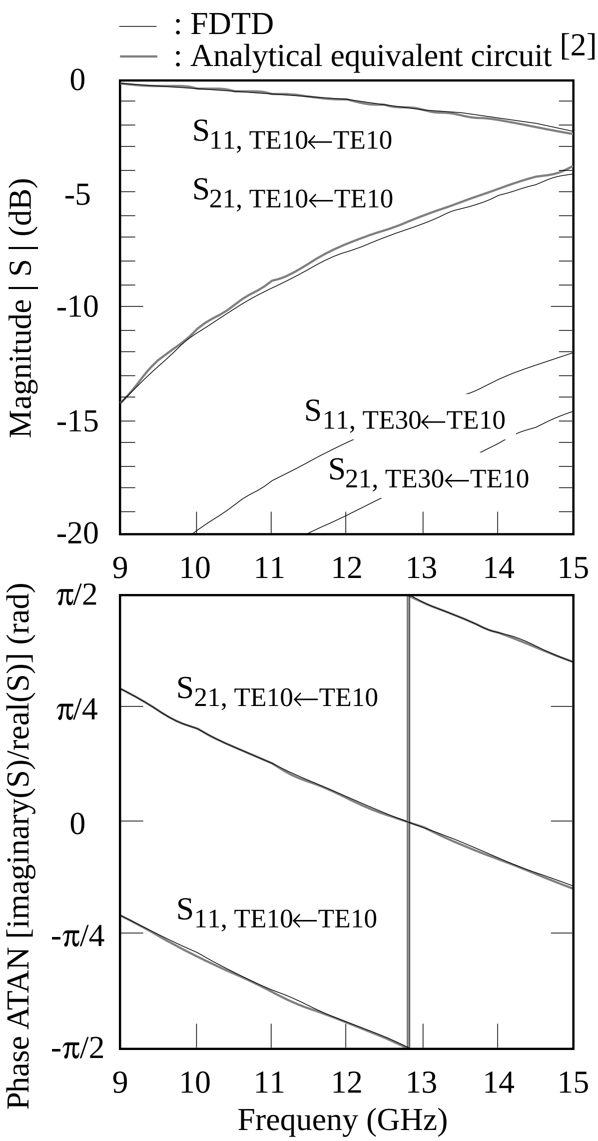

Figure 6 Scattering Parameters